16th U.S.-Japan-N.Z. Workshop
on the Improvement of Structural Engineering and Resiliency

MODAL DECOMPOSITION AND BEHAVIOUR OF FREE VIBRATION RESPONSE WITH GROUNDING AND UPLIFTING

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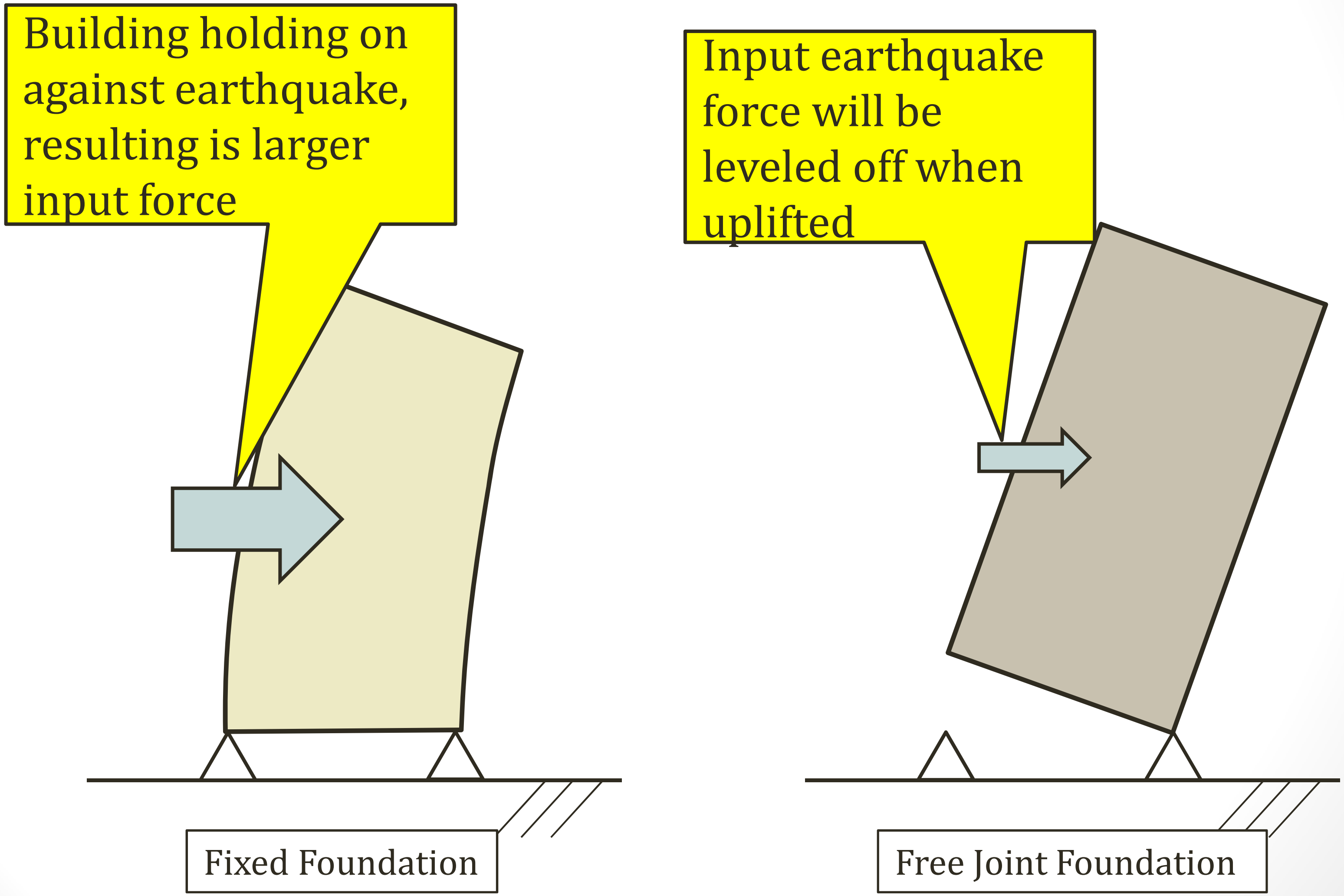
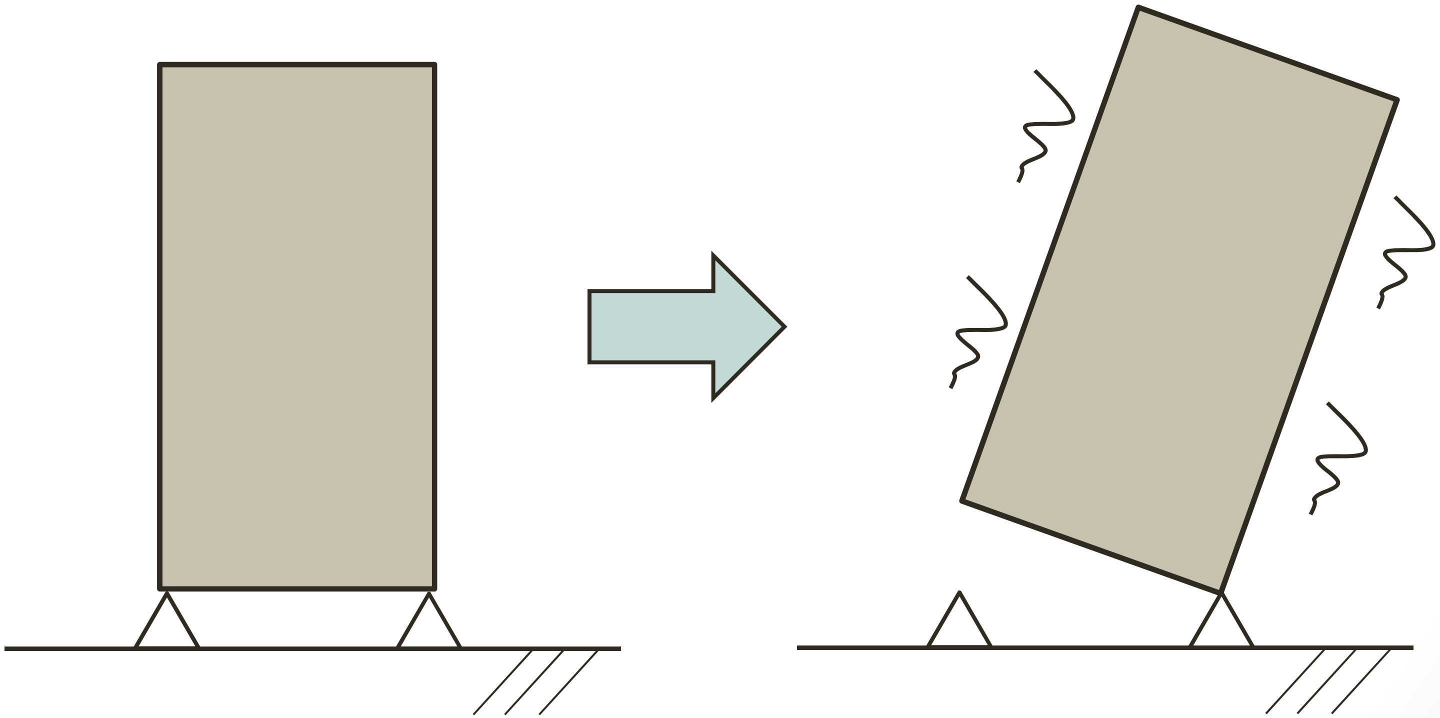
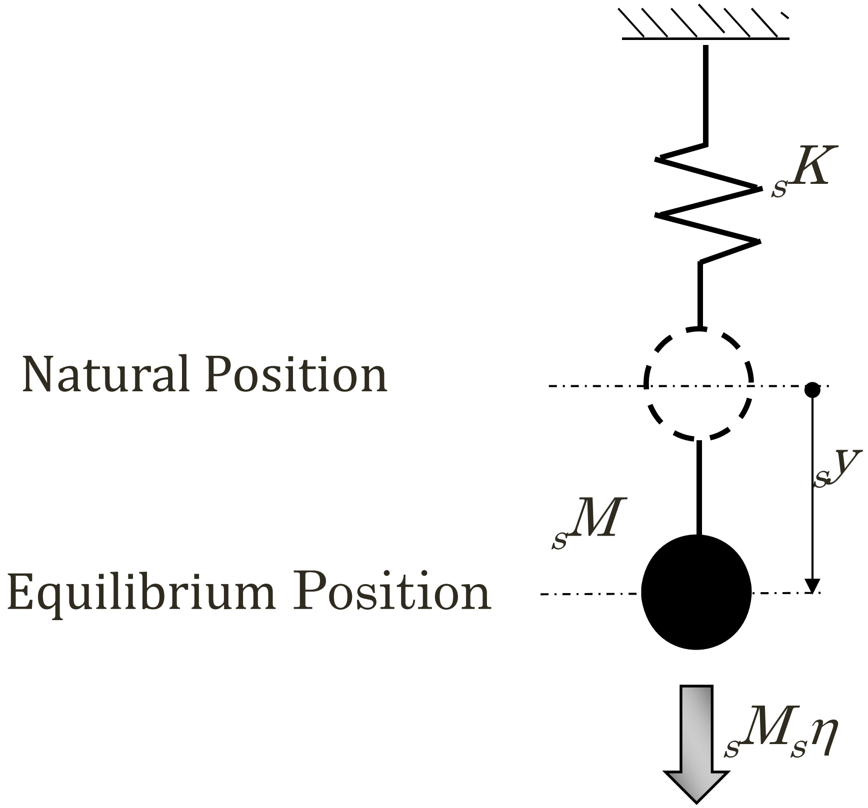
Uplifted Structure: Limiting earthquake force



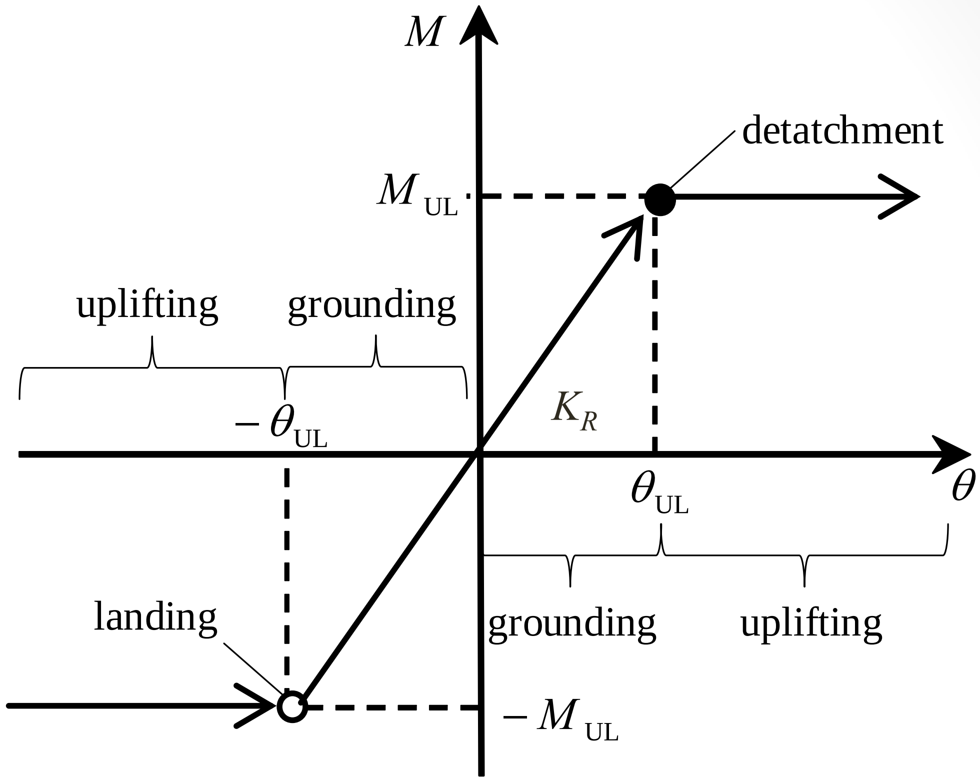
“Unknown random vibration” amplified and acting on the uplifted structure



Subject: Mechanism of Higher Mode Vibration



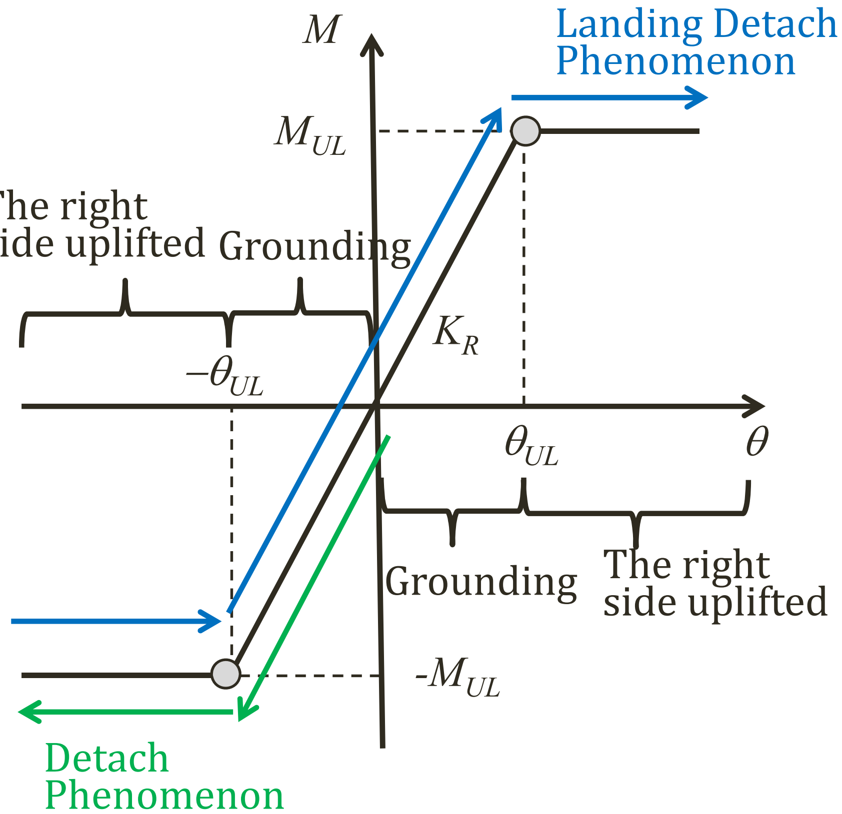
Spring Pendulum



Restoring force of Ground

- Uplifting
- Landing
- Grounding
- Detaching
- Uplifting Ultimate Moment
- Rocking Stiffness

- Detach Phenomenon
- Landing Detach Phenomenon

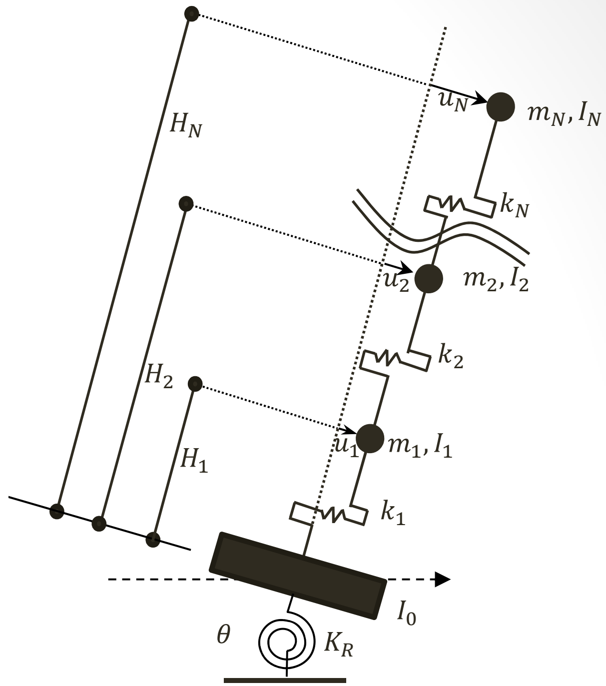


Restoring force characteristics
of ground spring R

Transposition of Non-linear Component

Equation of rocking motion model

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \boxed{\mathbf{f}_{\mathbf{ex}}} = \mathbf{0}$$



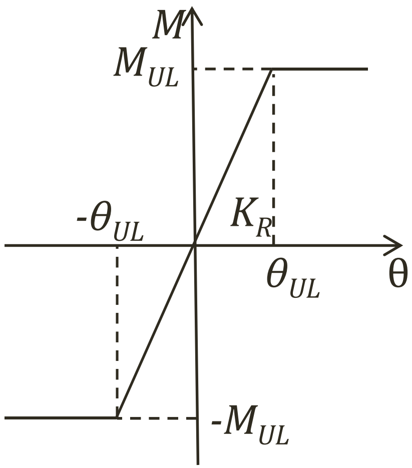
$$\mathbf{x} = \begin{Bmatrix} u_N \\ \vdots \\ u_1 \\ \hline \theta \end{Bmatrix} \qquad \mathbf{f}_{\mathbf{ex}} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ \hline M_R \end{Bmatrix}$$

$\cdot\ \cdot$ DOF of superstructure

$\cdot\ \cdot$ DOF of rocking

$$\mathbf{M} = \left[\begin{array}{ccc|c} m_N & & & m_N H_N \\ & \ddots & & \vdots \\ & & m_1 & m_1 H_1 \\ \hline & sym & & \sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^{\,2} \end{array}\right]$$

$$M_{\mathrm{R}} = \begin{cases} K_{\mathrm{R}}\theta & |\theta| \le \theta_{\mathrm{UL}} \\ M_{\mathrm{UL}} & |\theta| \ge \theta_{\mathrm{UL}} \end{cases}$$



$$\mathbf{K} = \left[\begin{array}{ccc|c} k_N & -k_N & & 0 \\ -k_N & \ddots & -k_2 & \vdots \\ & -k_2 & k_2 + k_1 & 0 \\ \hline & sym & & 0 \end{array}\right]$$

Transposition of Non-linear Component

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} \boxed{+\mathbf{f}_{\mathbf{ex}}} = \mathbf{0}$$

Rocking reaction force

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \boxed{-\mathbf{f}_{\mathbf{ex}}}$$

$\mathbf{f}_{\mathbf{ex}}$ can be expressed as multiplication of M and $\ddot{\mathbf{x}}_{\mathbf{ex}}$

$$-\mathbf{f}_{\mathbf{ex}} = \left\{ \begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline -M_{\mathrm{R}} \end{array} \right\} = \mathbf{M} \left\{ \begin{array}{c} H_N \\ \vdots \\ H_1 \\ \hline -1 \end{array} \right\} \ddot{\theta}_{\mathrm{ex}} = \mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}}$$

Where $\ddot{\theta}_{\mathrm{ex}} = M_{\mathrm{R}} \Big/ \sum_{i=0}^{N} I_i$ ・・rocking gravity acceleration

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}}}$$

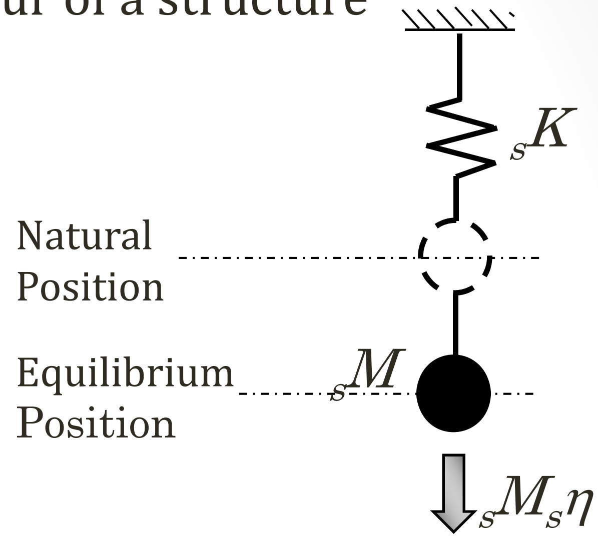
Modal Decomposition

Expresses uplifting and grounding behaviour of a structure

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}}$$

$\mathbf{x}$: Structural response

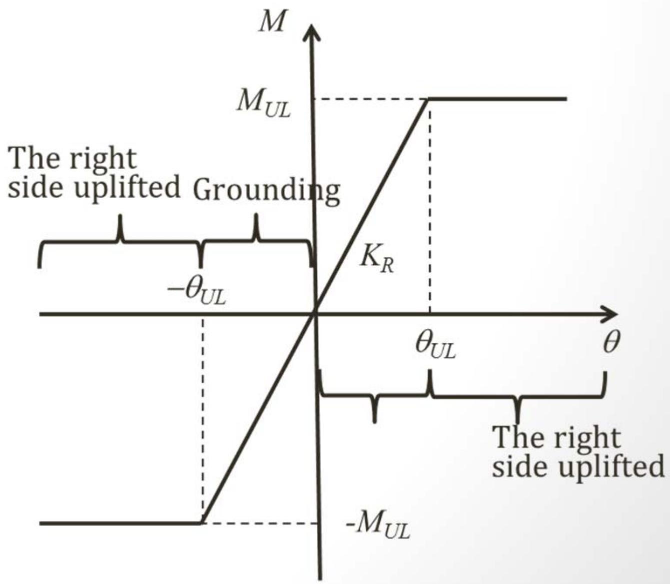
$\ddot{\mathbf{x}}_{\mathbf{ex}}$: gravity acceleration vector



$${}_sM\,{}_s\ddot{y} + {}_sC\,{}_s\dot{y} + {}_sK\,{}_sy = {}_sM\,{}_s\eta$$

${}_s\eta$:Generalized gravity acceleration

${}_sM_s\eta$: Generalized gravity



Rigid Rotation 1st Mode

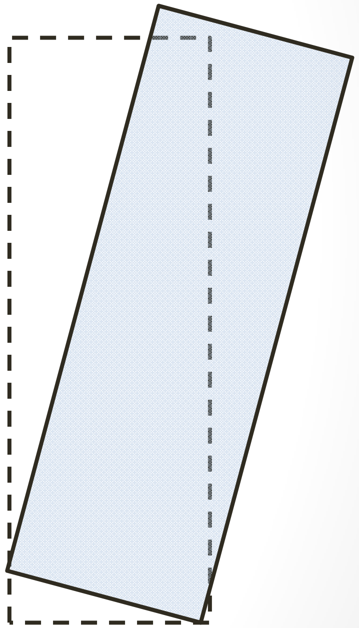
$${}_s\Omega \mathbf{M}\,{}_s\boldsymbol{\varphi} + \mathbf{K}\,{}_s\boldsymbol{\varphi} = \mathbf{0} \qquad s = 1,2,\cdots N+1$$

$$\mathbf{M} = \left[\begin{array}{ccc|c} m_N & & & m_N H_N \\ & \ddots & & \vdots \\ & & m_1 & m_1 H_1 \\ \hline sym & & & \sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^2 \end{array}\right] \qquad \mathbf{K} = \left[\begin{array}{cccc|c} k_N & -k_N & & & 0 \\ -k_N & \ddots & & -k_2 & \vdots \\ & & -k_2 & k_2 + k_1 & 0 \\ \hline sym & & & & 0 \end{array}\right]$$

$${}_1\Omega = 0$$

$${}_1\boldsymbol{\varphi} = {}^t\{0 \quad \cdots \quad 0 \mid {}_1\lambda\} \qquad {}_1\lambda : Arbitrary$$

→1st mode is rigid rotation mode



$${}_1K = {}_1^t\boldsymbol{\varphi}\mathbf{K}\,{}_1\boldsymbol{\varphi} = 0$$

$${}_1C = 2\,{}_1h\sqrt{{}_1M\,{}_1K} = 0$$

→Generalized Stiffness, Damping will be 0

$${}_1M\,{}_1\ddot{y} = {}_1M\,{}_1\eta \quad \Rightarrow \quad {}_1\ddot{y} = {}_1\eta$$

→1st mode behaviour is uniform acceleration

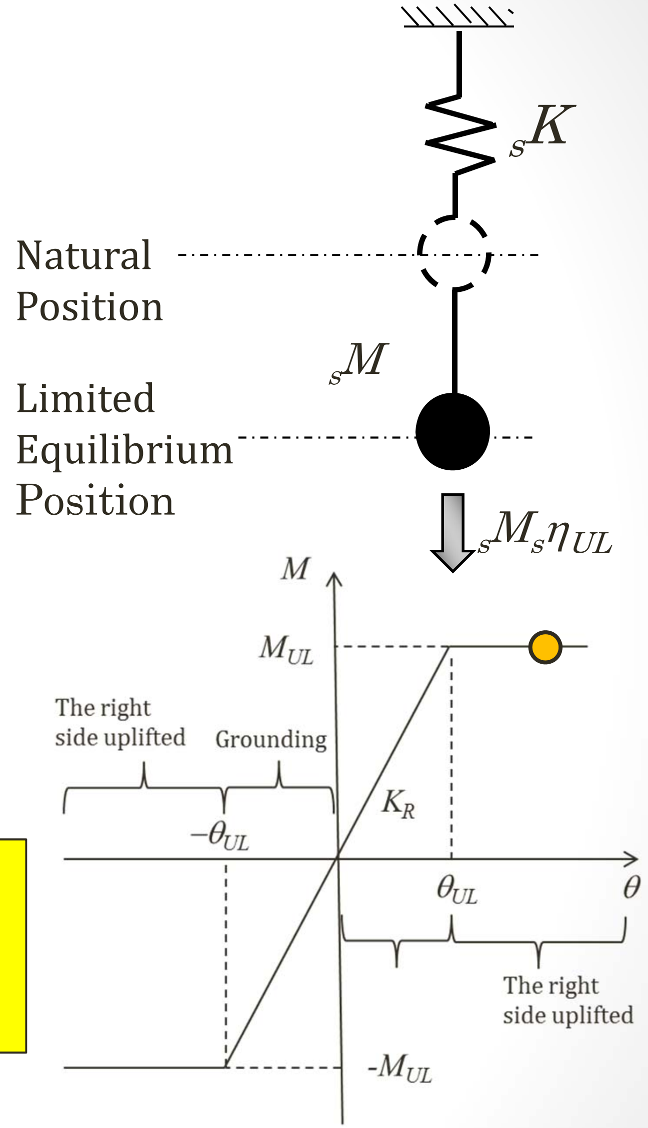
Limited Equilibrium Position

$$ {}_sM_s\ddot{y} + {}_sC_s\dot{y} + {}_sK_sy = {}_sM_s\eta_{UL} $$

Rotational restoring force is constant
$M = M_{UL}$ at uplifting phase.
Hence, generalized gravity and
equilibrium position will be constant

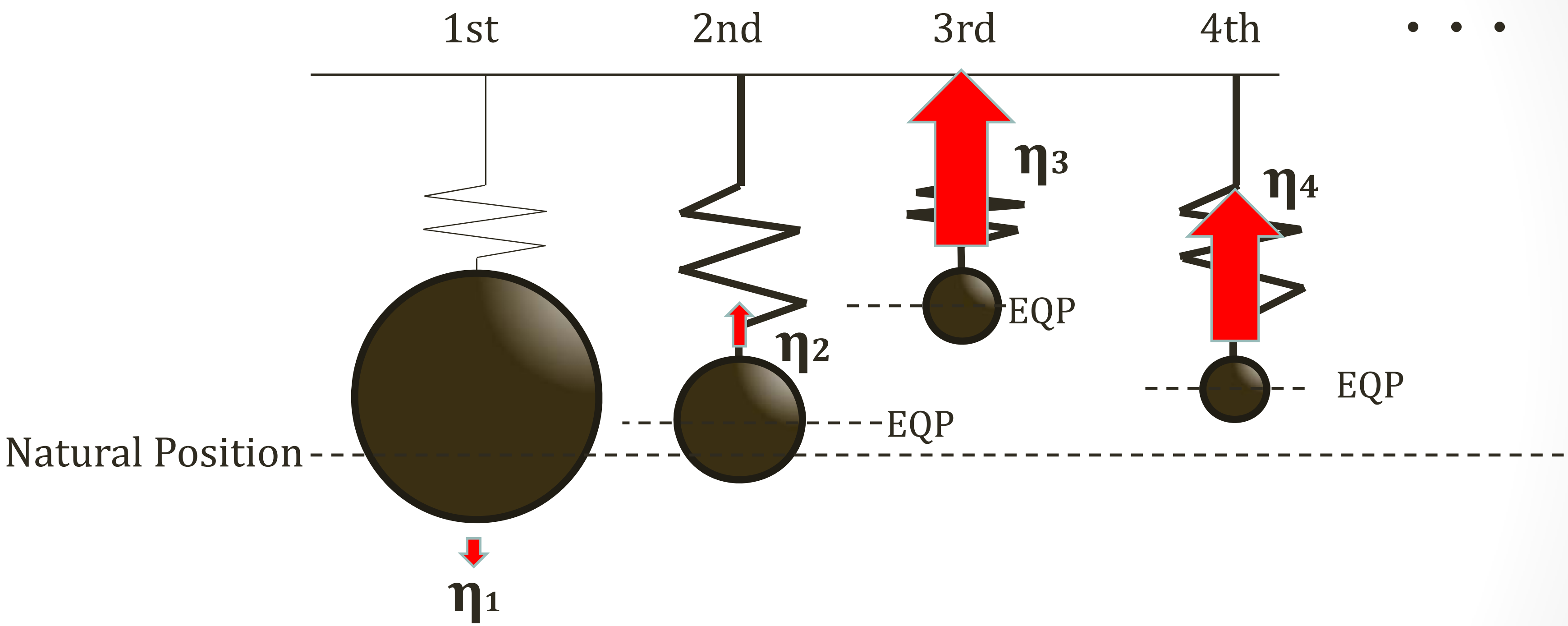
$$ {}_sK\,{}_sy_{gUL} = {}_sM_s\eta_{UL} \quad \Rightarrow \quad {}_sy_{gUL} = \frac{{}_sM_s\eta_{UL}}{{}_sK} $$

S^{th} mode spring pendulum will oscillate about limited equilibrium position and will be stopped at y_{gUL} after fully attenuated



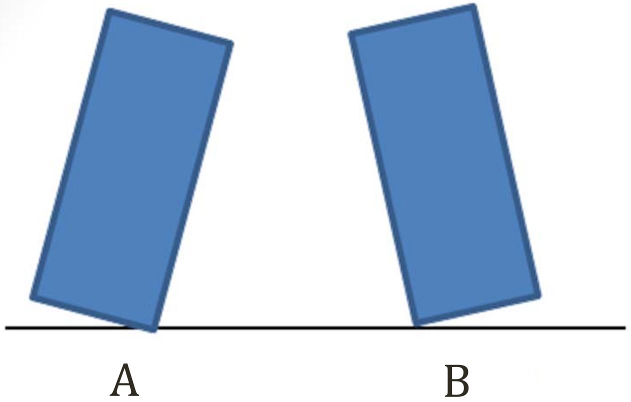
Limited Equilibrium Position

Spring Pendulum of each mode at static equilibrium position

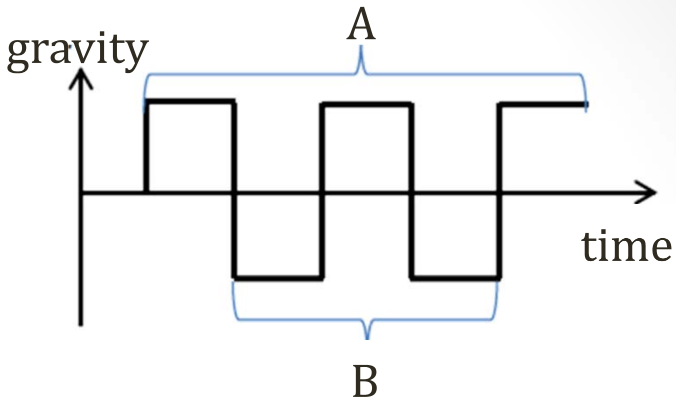


EQP: Equilibrium Position

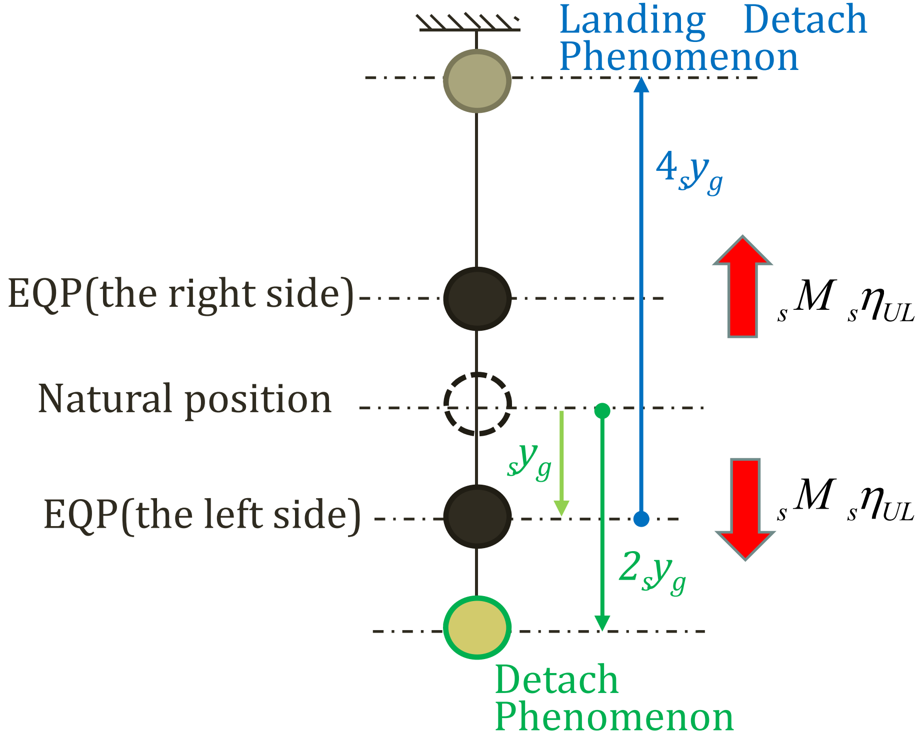
Mechanism of Higher Mode Vibration



Uplifting Phase



Change in the direction of gravity on each mode



Spring pendulum of each mode

Mechanism of Higher Mode Vibration

For Each Mode